Influence of atmospheric turbulence on planetary transceiver laser ranging

D. Dirkx[1], *R. Noomen*[1], *I. Prochazka*[2], *S. Bauer*[3], *L.L.A. Vermeersen*[1]

1: Delft University of Technology, The Netherlands
2: Czech Technical University in Prague, Czech Republic
3: DLR Berlin, Germany

Abstract

We investigate the influence of atmospheric turbulence on the performance of the uplink of a planetary transceiver laser ranging system using a single photon detector. We numerically combine the influence of turbulence in the mean intensity profile variations, scintillation, beam-wander induced pointing errors and stochastic time-of-flight variations. Thereby, we map the intensity variations due to turbulence to variations in the probability distribution of the arrival time of the 1st photon in a laser pulse. The turbulence models are applied to assess the influence on single-pass range accuracy and precision statistics, as well as the parameter estimation quality of a Phobos Laser Ranging (PLR) mission.

The difference in range measurement error between weak and strong turbulence is 3-4 mm in a PLR concept. This indicates that turbulence is a potentially important contributor to the error budget of interplanetary laser ranging missions, which aim at mm-level accuracy and precision. The single-shot precision is generally weakly influenced by turbulence, but strong turbulence is found to cause a strong decrease in detected pulse fraction, reducing normal point precision. We show that a trade-off between range accuracy and precision must be made when selecting laser system parameters, which is influenced by atmospheric turbulence effects.

We perform parameter estimations of Phobos initial state and observation biases using simulated measurements with and without turbulence, using a daily periodic turbulence strength model. We show that the parameter estimation quality is degraded significantly below that of the turbulence-free case only in the presence of strong turbulence.

1 Introduction

In the Satellite Laser Ranging (SLR), the influence of turbulence-induced stochastic time-of-flight variations has been studied by Kral et al. (2005), who verified the validity of the model derived by Gardner (1976) from measurements at the Graz SLR station. The influence of scintillation on the performance of SLR systems has received more limited attention. However, the statistical influence of (time-dependent) turbulence-induced signal-intensity variations on system performance has not been quantified in detail to date.

One- and two-way active transceiver laser ranging systems (Degnan, 2002) are an emerging technology that is based on existing SLR and LLR technology, modified with an active space segment to allow larger distances to be covered. These technologies have the potential to deliver mm-precise measurements over interplanetary distances, extending the technology of SLR and LLR to Interplanetary Laser Ranging (ILR). The increased range precision and accuracy that can be obtained, compared to current radiometric systems, are expected to yield order(s) of magnitude improvements in the estimation of science parameters related to, for instance, gravitational physics (Turyshev et al., 2010) and planetary interiors (Dirkx et al., 2014b).

The measurement error budget breakdown of ILR systems will be different from that of SLR and LLR. The role of optical turbulence in ILR has not yet been assessed. Quantification of the various error sources of ILR will be crucial in setting up system requirements during conceptual mission design, as well as for assessing the potential science return from missions using this technology. As opposed to SLR, where ranging data is freely and widely available, no such data exists for ILR, so that we are forced to rely on simulated data for performing analyses of the expected system performance.

Here, we investigate the influence of optical turbulence on the range precision and accuracy of the uplink of an ILR system (*i.e.* Earth-to-space). We limit ourselves to the uplink of the system for several reasons. Firstly, aperture averaging is expected to reduce the scintillation effects for the downlink (Degnan, 1995). Secondly, the far-field (as opposed to near-field) turbulence in the case of the downlink cause effects such as beam wander and beam spread to be (nearly) absent.

This paper, including this introduction, is based heavily on the paper by Dirkx et al. (2014a) and can be seen as a concise summary of the work presented there. Here, we focus on the key results regarding the influence of turbulence, deferring many of the details to the main paper. We first give an overview of the models used in our simulations in Section 2, followed by our main results on the influence of turbulence on accuracy and precision statistics in Section 3 and our overall conclusions in Section 4.

2 Model summary

In this section, we will give a stepwise overview of our numerical procedure to combine the models presented in the previous sections to generate range measurement statistics.

- Calculate from a given pulse transmission time the measurement geometry for an ideal link, calculating the ideal reception time and the state of both the receiver and transmitter at reception and transmission time. From the geometry we obtain the link distance z and zenith angle ζ.

- Evaluate the model for σ_p^2, which quantifies the beam-wander induced pointing error. Use σ_P^2 to generate realizations of the Gaussian PDFs for the two pointing errors from which we obtain an off-axis target distance r.

- Calculate the mean pulse intensity at the target $\langle I(r,z)\rangle$, using models for the long-term spot size, system parameters and current values of r and z. We use zenith angle ζ and atmospheric transmittance ($T_a = 0.7$), as well as the transmission system efficiency η_t

- Evaluate the model for the scintillation index σ_I^2.

- Generate a realization of the intensity at the receiver from the Gamma distribution using σ_I^2 and $\langle I(r,z)\rangle$.

Table 1: Nominal parameters for Earth-Phobos laser link.

λ	$U_{p,t}$	t_{FWHM}	θ_{FF}	r_d	$\eta_q \cdot \eta_r$
532 nm	1.0 mJ	50 ps	25 μrad	0.25 m	0.12

- From the total intensity at the detector and the receiving telescope diameter, obtain the total energy that is incident on the detectot. Generate a realization of the ideal number of detectable photons N_{id} (*i.e.* from the Poisson distribution.
- Use the binomial distribution with N_{id} possibly detectable photons with equal and independent possibility of detection $\eta_r\eta_q$ to determine number of detected photons N, where η_r is the receiver optical efficiency and η_q is the detector quantum efficiency.
- From N and the pulse length σ_t, generate a realization of the pulse detection time error; add stochastic time of flight variation..

The primary output of this procedure is the pulse measurement time error τ.

3 Results: range measurement statistics

In this section, a summary of the simulation results for the influence of atmospheric turbulence on the range measurements statistics of a representative mission and system are presented and discussed. We investigate the influence of time-invariant turbulence on the range accuracy and precision statistics over a single pass. We use the Huffnagel-Valley (HV) model for the influence of turbulence, which is parameterized by $C^2_{n(0)}$ and u, representing the ground level turbulence strength, and mean high-altitude winds, respectively.

In Section 3.1, we outline our simulation scenario and setup, discussing the mission parameters that we use. Subsequently, we discuss the influence of turbulence on the measurement accuracy and precision in Sections 3.2 and 3.3, respectively. We defer the results of an estimation with a turbulence model in-the-loop to Dirkx et al. (2014a).

3.1 Simulation parameters

We use the uplink of the Phobos Laser Ranging (PLR) system of Turyshev et al. (2010) as the test case for our analysis, and use similar system parameters as nominal input to our simulations, shown here in Table 1. Here $U_{p,t}$ denotes the transmitted pulse energy, which includes the transmission efficiency η_t. We choose to analyze a system of intermediate signal strength (nominally operating at the low multi-photon levels), as such systems are not overdesigned from a system power point-of-view, while retaining a comfortable margin of allowable signal strength degradation. Also, simulations of such a system provide us with insight into both the degradation in system accuracy due to pulses with multiple detectable photons, which will occur under weak turbulence strength conditions, as well as normal point precision degradation due to a reduction in the number of detections, which will occur under strong turbulence conditions.

We investigate a broad range of values for $C^2_{n(0)}$, with 0 m/s $\leq u \leq$ 50 m/s and $2.0 \cdot 10^{-16}$ m$^{-2/3}$ $\leq C^2_{n(0)} \leq 10^{-11}$ m$^{-2/3}$. It should be noted that the $> 10^{-12}$ m$^{-2/3}$ turbulence case represents rather extreme turbulence conditions. Such large ground turbulence values are only expected at daytime.

Table 2: Nominal measurement statistics for turbulence-free Earth-Phobos laser link.

$\overline{\Delta s}$	$\sigma_{\Delta s}$	Detected fraction	$\overline{N}$	σ_N
-4.93 mm	5.66 mm	94.35 %	3.09	1.87

3.2 Range accuracy

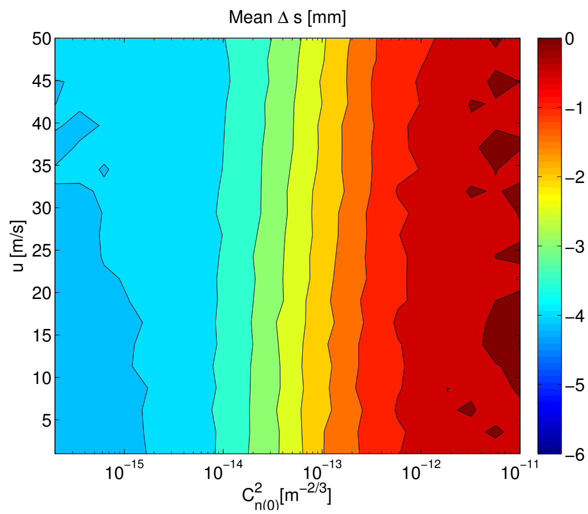
The results for the mean range errors (*i.e.* accuracy) as a function of the HV parameters $C^2_{n(0)}$ and u are shown in Fig. 1(a). It can be seen that the range error is mostly influenced by the ground turbulence term $C^2_{n(0)}$ and much less by the high-altitude wind speed u, indicating a weak dependence of accuracy on the scintillation index σ^2_I).

For strong turbulence conditions, the mean range error reaches values of nearly 0 mm, as shown in Fig. 1(a). This indicates that nearly all detections occur for cases where only a single photon reaches the detector. However, it is interesting to note that even for very low energy levels, detections still occur for which $\Delta s \neq 0$, so where $N > 1$. This is due to the Poisson statistics describing the ideal distribution of N. Relatedly, consistently operating at or near the single photon energy level will mitigate most of the accuracy degradations due to the turbulence (or other sources of varying received signal intensity) and is therefor highly recommended

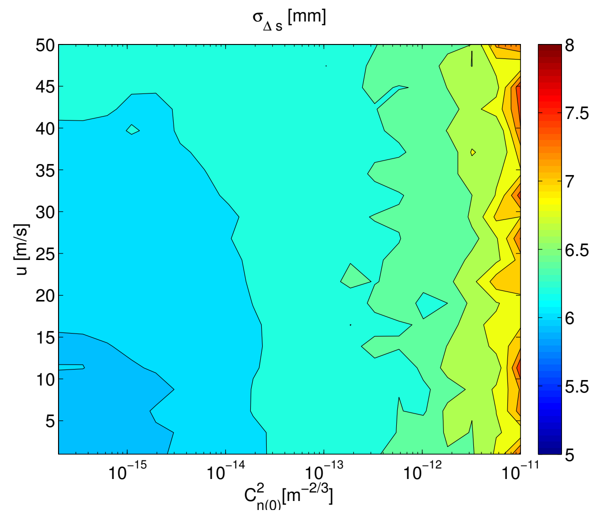
The lack of smoothness in the plot at large $C^2_{n(0)}$ is due to the very small number of detected pulses under these strong turbulence conditions. The small number of data points used to numerically calculate the mean of the range error causes deviations from the ideal mean range error. For the low values of $C^2_{n(0)}$, the number of pulse detection is substantially higher, leading to a more robust determination of the mean range error.

The mean value of the range error can be seen in Fig. 1(a) to vary over a range of roughly 3-4 mm between low ($< 10^{-15}$ m$^{-2/3}$) and high ($> 10^{-12}$ m$^{-2/3}$) $C^2_{n(0)}$, with very little variation due to variations in u. The larger range error for weak turbulence conditions is due to the higher average value of N in these cases, leading to a detection earlier in the laser pulse. The observed range error variations between weak and strong turbulence conditions are well above the sub-mm level that are desired for interplanetary laser ranging. This indicates that time variations in the ground turbulence strength at even the moderate variation of 10^{-15} – 10^{-13} m$^{-2/3}$ could cause a noticeable degradation of ILR system performance. The estimation of range biases, including those resulting from the influence turbulence are typically estimated during data processing. However, time-variabilities of range biases may be difficult to remove during data analysis and could degrade estimation performance (Dirkx et al., 2014a).

The reduction in signal strength at the receiver (*i.e.* smaller value of N) due to large $C^2_{n(0)}$ is primarily a result of the strong increase in pointing error $\Delta\theta$ in strong turbulence conditions. Both the mean value and standard deviation of the pointing error grow above values of 100 μrad ($= 4\theta_{FF}$, see Table 1) for $C^2_{n(0)} > 10^{-12}$ m$^{-2/3}$. Mitigation strategies for pointing-error induced signal strength reduction are discussed by Dirkx et al. (2014a). Since the beam-wander induced variations are the dominant source of strong accuracy variations, our approach of analyzing only the uplink of the two-way laser ranging system is a valid one for this conceptual analysis, since beam wander is not present in the downlink.



(a)



(b)

Figure 1: Results for nominal mission scenario and varying turbulence conditions of a) mean range error and b) standard deviation of range error of detected pulses.

3.3 Range precision

The standard deviation (single-shot precision) of the pulse detection times (*i.e.* precision) as a function of turbulence strength is shown in Fig. 1(b). For the nominal pulse length of 50 ps FWHM used here (Table 1), the impact of turbulence on the precision of the measurements is relatively small. We find a variation of about 1-1.5 mm single-shot precision between weak and strong turbulence conditions, compared to a turbulence-free value of about 5.6 mm single-shot precision (Table 2).

The behaviour of the range precision with turbulence parameters in Fig. 1(b) exhibits a moderately stronger dependence on u than the range accuracy (see Fig. 1(a)). This is due to the influence of scintillation on the range precision. However, for larger values of $C^2_{n(0)}$, the variations in range measurement precision are no longer noticeably dependent on u, and are physically dominated by the strong variations in pointing angle error $\Delta\theta$, as was the case for the range accuracy.

Although the influence on single-shot precision is quite small, we do find a very strong decrease in the detected pulse fraction between weak and strong turbulence. This decrease is due to the large turbulence-induced pointing error in strong turbulence. As a result, although the single-shot precision is left largely unaffected by turbulence, the normal point precision for a given time interval decreases, or alternatively the time to reach a certain normal point precision increases, because fewer pulses are detected per unit time.

4 Conclusions

We have presented the results of simulations to analyze the influence of atmospheric turbulence on the performance of planetary laser ranging systems, using a Phobos Laser Ranging mission as representative test case. We have taken mean intensity profile variations, scintillation, beam-wander induced pointing errors and stochastic time-of-flight variations into account. Using the Hufnagel-Valley turbulence profile model, we calculated the influence of turbulence on range accuracy and precision as well as parameter estimation performance.

We find that for our mission test case, turbulence-induced signal strength variations cause a variation in range error of 3-4 mm between weak and strong turbulence conditions. Nearly all strong accuracy variations are due to variations in ground turbulence strength $C^2_{n(0)}$, with little to no influence of the mean wind velocity. The magnitude of the turbulence-induced variations are at a level where they could be a significant contributor to an ILR error budget, which aims at sub-mm range accuracy. Influence of turbulence on single-shot precision is relatively small, at about 1 mm increase, compared to a nominal value of 5.6 mm. However, strong turbulence conditions cause a strong decrease in the detected pulse fraction, reducing the number of pulses that can be used to generate a normal point. The primary contributor to turbulence-induced accuracy and detected pulse fraction variations is found to be the turbulence-induced pointing error.

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