13-Po14

Development of software for high-precision LLR data analysis

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Abstract

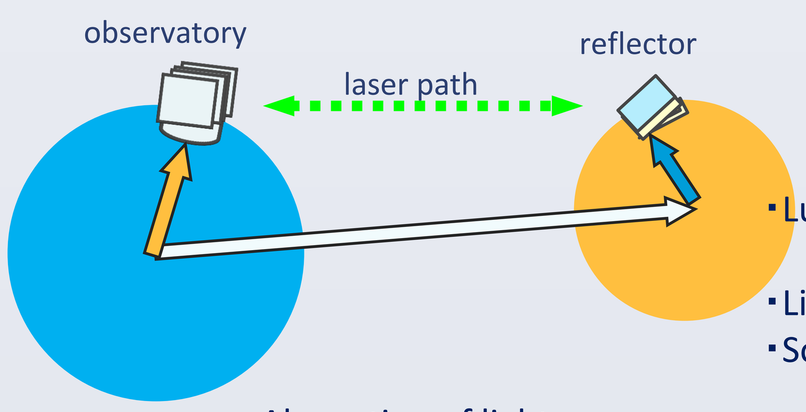
In order to determine the lunar orbital/rotational motion and tidal deformation using lunar laser ranging (LLR) observation data, analysis software is being developed. As the first step of the study, we construct an LLR observation model, combining the newest physical models.

The observation model consists of the lunar orbit and libration obtained from DE430 (provided by NASA JPL), and the other physical models compatible with IERS Conventions (2010) such as Earth orientation, solid Earth/Moon tides, and some factors affecting propagation delay. For the purpose of calculating these components precisely, we use the modules of the geodetic data analysis software "c5++" (Otsubo et al., JpGU, 2011). LLR observation data are provided as normal points obtained at Apache Point, Grasse, Matera and McDonald. In this calculation, there are 3372 normal points distributed from June 1996 to August 2012. Comparing the observed and calculated one-way range, the mean and the standard deviation of the residuals are about 5.7 cm and 4.8 cm respectively.

3 steps of this study	Current status
Step 1: **LLR observation modeling**	✓ **Almost finished (this report)**
Step 2: Lunar orbit/rotation integration	Unfinished
Step 3: Lunar orbit/rotation determination	Not yet started

Components of the Model

- Station coordinates
- Earth rotation
  - Earth rotation angle
  - Precession ∼ 100 m
  - UT1 - UTC ∼ 100 m
  - Nutation ∼ 10 m
  - Polar motion ∼ 10 m
- Solid Earth tides ∼ 0.1 m



- Lunar orbit around the Earth
- Libration
- Solid Moon tide ∼ 0.1 m

- Aberration of light ∼ 1 m
- Tropospheric delay ∼ 1 m
- Relativistic propagation delay ∼ 1 m
- 4-dim. space-time transformation ∼ 1 m (Lorentz transformation b/w TDB and TT)

(orders of magnitudes on one-way range)

(c5++: models calculated by using modules of "c5++")

Lunar orbit around the Earth, Librations (c5++)
- JPL lunar and planetary ephemeris DE430

Station coordinates and velocities (c5++)
- ITRF 2008 + Station eccentricities (∼1m)
- Apache Point Obs. Coordinates and velocities (by Prof. Müller, personal communication, 2013)

Solid Earth tide (c5++)
- IERS Conv. (2010) model

Earth rotation (c5++)
- IERS Conv. (2010) models + EOP 08 C04 data
  - ERA, precession, nutation, polar motion, UT1 – UTC correction

Reflector coordinates
- Williams et al. (2013)

Solid Moon tide
- Murphy et al. (2009)

$$\Delta \boldsymbol{r} = H\left(\frac{R_0}{R}\right)^3 \left\{ h_2 \hat{\boldsymbol{r}} \left[\frac{3}{2}(\hat{\boldsymbol{R}}\cdot\hat{\boldsymbol{r}})^2 - \frac{1}{2}\right] + 3 l_2 (\hat{\boldsymbol{R}}\cdot\hat{\boldsymbol{r}})\left[\hat{\boldsymbol{R}} - (\hat{\boldsymbol{R}}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}\right] \right\}$$

- Love numbers from Williams et al. (2013)
  $h_2 = 0.0476,\ l_2 = 0.0107$

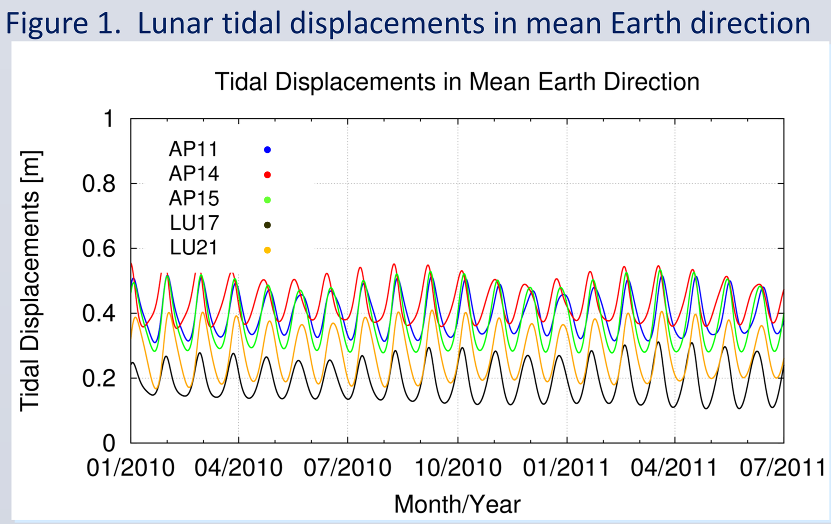


Figure 1. Lunar tidal displacements in mean Earth direction

Aberration of laser light
- Fukushima (ed.) (2009)
- Solving the equation of light time iteratively, using Newton method

Tropospheric propagation delay (c5++)
- Mendes et al. (2002) model

Relativistic propagation delay
- IERS Conv. (2010) Chapter. 11

$$c(t_2 - t_1) - r_{12} = (1+\gamma)\sum_{J=1}^{N}\left(\frac{GM_J}{c^2}\right)\ln\left|\frac{r_{1J}+r_{2J}+r_{12}}{r_{1J}+r_{2J}-r_{12}}\right|$$

- Effects of perturbers on one-way range: Sun ≈ 7 - 8 [m], Earth ≈ 30 - 40 [cm], Moon ∼ 0.1 [mm]

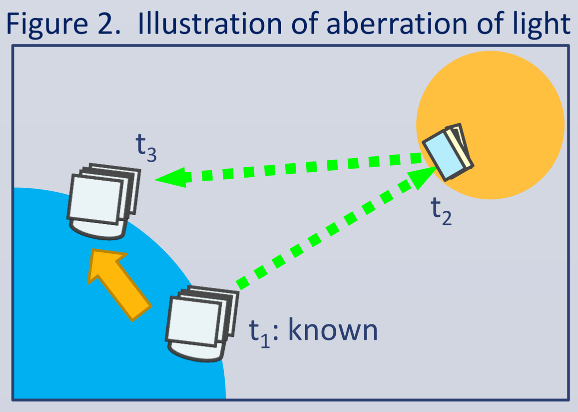


Figure 2. Illustration of aberration of light

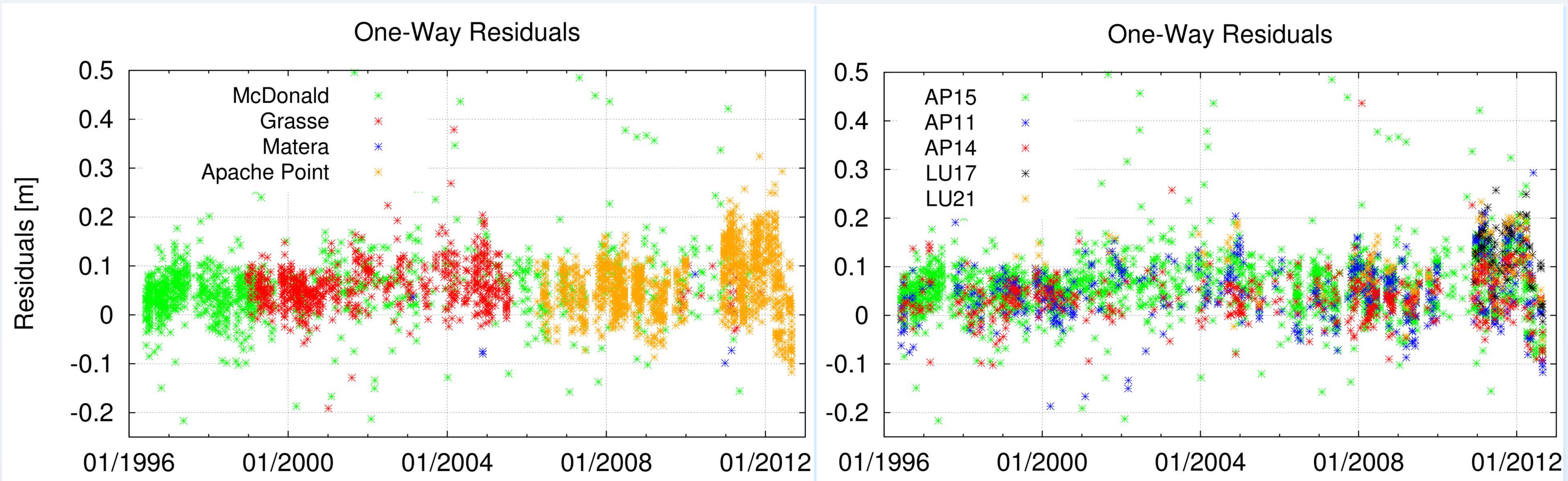
TDB – TT (time scale transformation)
- Kovalevsky et al. (1989)
- Needed to compensate for the difference in time scales between ephemeris and observed round-trip time
  - Ephemeris DE430: Barycentric Dynamical Time (TDB)
  - Round-trip time: Terrestrial Time (TT)
    - TT = UTC + 32.184 sec + accumulated Leap Seconds

Lorentz transformation (spatial components)
- IERS Conv. (2010) Chapter. 11
- Spatial transformation accompanied by time transformation TDB – TT

$$\boldsymbol{r}_{TDB} = \boldsymbol{r}_{TT}\left(1 - \frac{U}{c^2} - L_C\right) - \frac{1}{2}\left(\frac{\boldsymbol{V}}{c^2}\cdot\boldsymbol{r}_{TT}\right)\boldsymbol{V}$$

Results

Figure 3,4. One-way residuals (observed - calculated) by stations and reflectors



- 3372 normal points, June 1996 – August 2012 (data sources: CDDIS and APOLLO website)
- Mean: 5.7 cm, Standard deviation: 4.8 cm (with 11 % data rejected)

Table 1. Number of normal points we compared

	AP11	AP14	AP15	LU17	LU21
Apache Point	255	264	668	75	81
Grasse	135	126	418	0	25
Matera	1	1	10	0	0
McDonald	164	165	977	0	7

Figure 5. Residuals (Number of single raw ranges)

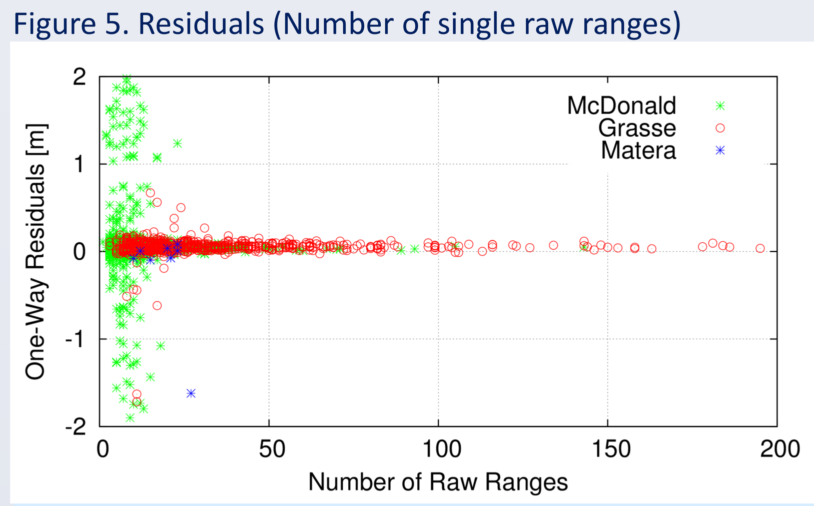


Figure 7. Comparison b/w two versions of ephemeris DE421 (2008) and DE430 (2013)

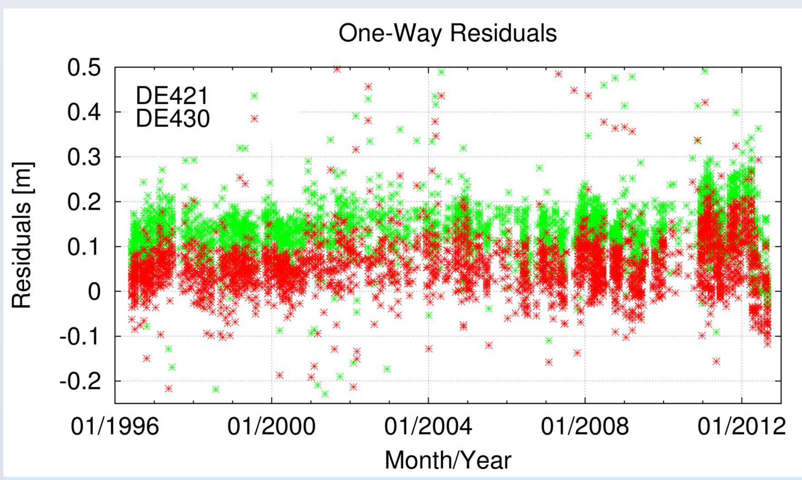
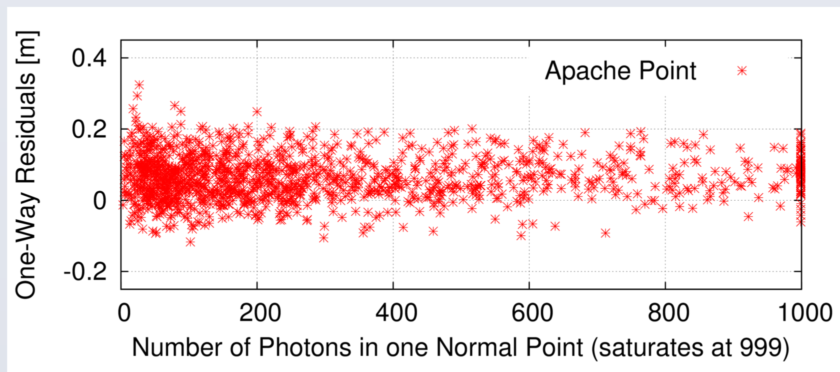


Figure 6. Residuals (Number of photons)

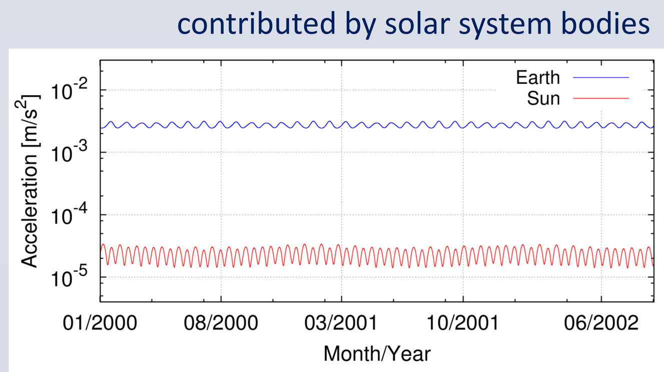


- Remaining factors (≲1 cm)
  → ocean tide loading, atmospheric loading, and target signatures
- Normal points observed before 1996 need to be considered

Next Step: Orbit Integration (preliminary)

Acceleration of the Moon as a point mass w.r.t. the Earth
- Point mass interactions
  - Non-relativistic

Figure 8,9. Absolute values of lunar acceleration w.r.t. Earth, contributed by solar system bodies (unit: [m/s^2], Calculated by using DE430)



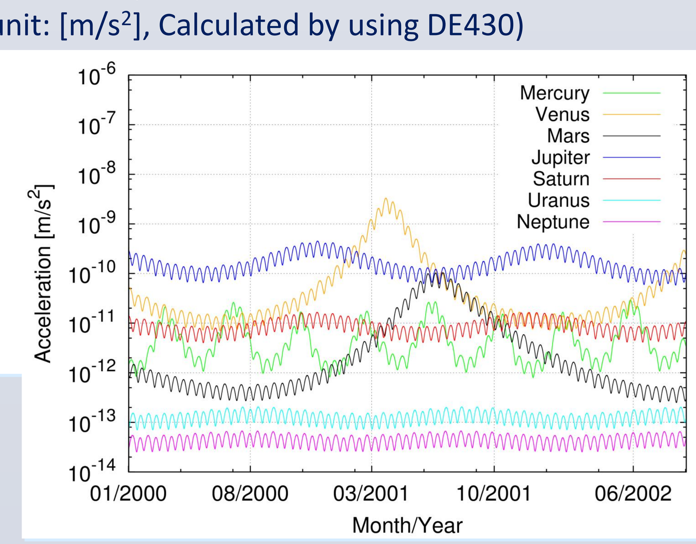
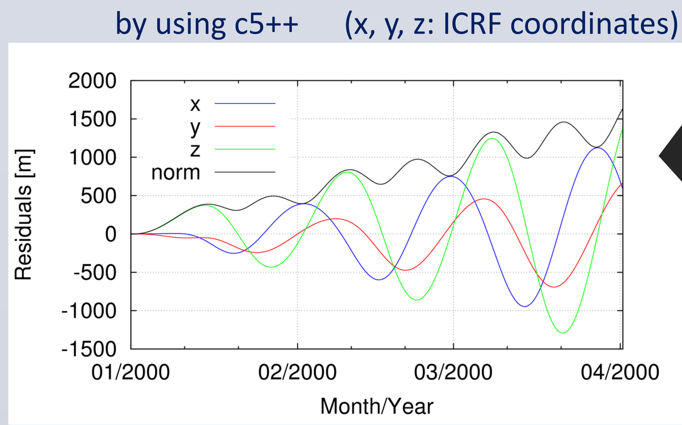


Figure 10. Residuals of lunar orbit around the Earth b/w DE430 and tentative orbit integrated by using c5++ (x, y, z: ICRF coordinates)



(c5++)
- Components: Sun, Solar system planets, Earth J_2 (very preliminary study)
- Cowell method, double-precision calculation
- Initial condition: fixed to DE430

- Relativistic (Einstein-Infeld-Hoffmann equation): ≲ 10e-10 m/s^2
- Figure effect of the Earth and the Sun : ≲ 10e-09 m/s^2 … mostly Earth J_2
- Earth tides effect : The tides raised upon the Earth affect the motion of the Moon

Acknowledgement

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